

Chapter 2

Reasoning and Proof

Section 1

Conditional Statements

GOAL 1: Recognizing Conditional Statements

In this lesson you will study a type of logical statement called a conditional statement.

A conditional statement has two parts, a hypothesis and a conclusion.

When the statement is written in if-then form, the “if” part contains the hypothesis and the “then” part contains the conclusion.

Here is an example:

If **it is noon in Georgia**, then **it is 9 A.M. in California**.

Hypothesis **Conclusion**

Example 1: Rewriting in If-Then Form

Rewrite the conditional statement in if-then form.

- a. Two points are collinear if they lie on the same line.

If two points lie on the same line, then they are collinear.

- a. All sharks have a boneless skeleton.

If it is a shark, then it has a boneless skeleton.

- a. A number divisible by 9 is also divisible by 3.

If a number is divisible by 9, then it is also divisible by 3.

Conditional statements can either be true or false. To show that a conditional statement is true, you must present an argument that the conclusion follows for *all* cases that fulfill the hypothesis. To show that a conditional statement is false, describe a single counterexample that shows the statement is not always true.

Example 2: Writing a Counterexample

Write a counterexample to show that the following conditional statement is false.

If $x^2 = 16$, then $x = 4$.

$$\text{C.E.} \rightarrow x = -4$$

The ____converse____ of a conditional statement is formed by switching the hypothesis and conclusion. Here is an example.

Statement: If you see lightning, then you hear thunder.

Converse: If you hear thunder, then you see lightning.

Example 3: Writing the Converse of a Conditional Statement

Write the converse of the following conditional statement.

Statement: If two segments **are congruent**, then they **have the same length**.

Converse: If two segments **have the same length**, then they **are congruent**.

A statement can be altered by ___negation___, that is, by writing the negative of the statement. Here are some examples.

STATEMENT

$$m\angle A = 30^\circ$$

$\angle A$ is acute.

NEGATION

$$m\angle A \neq 30^\circ$$

$\angle A$ is not acute.

When you negate the hypothesis and conclusion of a conditional statement, you form the ___inverse___. When you negate the hypothesis and conclusion of the converse of a conditional statement, you form the ___contrapositive___.

*contrapositive \rightarrow flip AND negate the conditional

Original	If $m\angle A = 30^\circ$, then $\angle A$ is acute.	<p>Both false Both true</p>
<i>negate</i> Inverse	If $m\angle A \neq 30^\circ$, then $\angle A$ is not acute.	
Converse	If $\angle A$ is acute, then $m\angle A = 30^\circ$.	
<i>flip flip & negate</i> Contrapositive	If $\angle A$ is not acute, then $m\angle A \neq 30^\circ$.	

When two statements are both true or both false, they are called ____equivalent statements____. A conditional statement is equivalent to its contrapositive. Similarly, the inverse and converse of any conditional statement are equivalent. This is show in the table above.

Example 4: Writing an Inverse, Converse, and Contrapositive

Write the (a) inverse, (b) converse, and (c) contrapositive of the statement.

If there is snow on the ground, then flowers are not in bloom.

Inverse (negate):

If there is not snow on the ground, then flowers are in bloom.

Converse (flip):

If flowers are not in bloom, then there is snow on the ground.

Contrapositive (flip & negate):

If flowers are in bloom, then there is not snow on the ground.

Goal 2: Using Point, Line, and Plane Postulates

In Chapter 1, you studied four postulates.

Ruler Postulate	(Lesson 1.3, page 17)
Segment Addition Postulate	(Lesson 1.3, page 18)
Protractor Postulate	(Lesson 1.4, page 27)
Angle Addition Postulate	(Lesson 1.4, page 27)

Remember that postulates are assumed to be true – they form the foundation on which other statements (called theorems) are built.

POINT, LINE, AND PLANE POSTULATES

POSTULATE 5 Through any two points there exists exactly one line.

POSTULATE 6 A line contains at least two points.

POSTULATE 7 If two lines intersect, then their intersection is exactly one point.

POSTULATE 8 Through any three noncollinear points there exists exactly one plane.

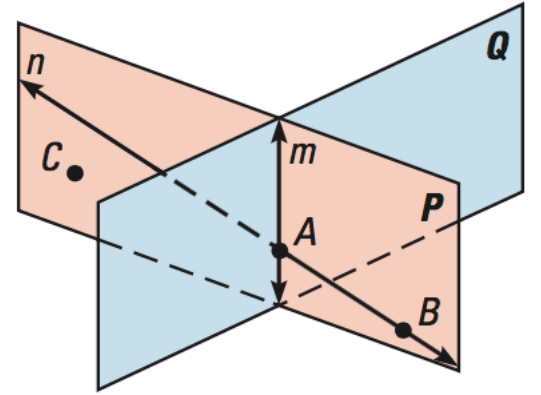
POSTULATE 9 A plane contains at least three noncollinear points.

POSTULATE 10 If two points lie in a plane, then the line containing them lies in the plane.

POSTULATE 11 If two planes intersect, then their intersection is a line.

Example 5: Identifying Postulates

Use the diagram at the right to give examples of Postulates 5 through 11.



Example 6: Rewriting a Postulate

a. Rewrite Postulate 5 in if-then form.

b. Write the inverse, converse, and contrapositive of Postulate 5.

inverse:

converse:

contrapositive:

Example 7: Using Postulates and Counterexamples

Decide whether the statement is true or false. If it is false, give a counterexample.

- a. A line can be in more than one plane.
- b. Four noncollinear points are always coplanar.
- c. Two nonintersecting lines can be noncoplanar.